Mark Scheme (Results)

January 2023

Pearson Edexcel International Advanced Level
In Pure Mathematics P3 (WMA13) Paper 01

Question Number	Scheme	Marks
1(a)	$\mathrm{f}(x) \leqslant 9$	B1
		(1)
(b)	$\mathrm{fg}(1.5) = \mathrm{f}\left(\dfrac{3}{2\times1.5+1}\right) = 9 - \left(\dfrac{3}{2\times1.5+1}\right)^2$	M1
	$=\dfrac{135}{16}$	A1
		(2)
(c)	$\mathrm{g}(x) = \dfrac{3}{2x+1} \Rightarrow \mathrm{g}^{-1}(x) = \dfrac{3-x}{2x}$	M1 A1
	$0 < x \leqslant 3$	B1
		(3)
		Total 6

(a)

B1: Correct range. Allow equivalent notation e.g. $y \leqslant 9$, $\mathrm{f} \leqslant 9$, $y \in (-\infty, 9]$ but not $x \leqslant 9$.

Condone just "$\leqslant 9$" and "less than or equal to 9"

(b)

M1: Full attempt at method to find fg (1.5) condoning slips. Implied by a correct answer or 8.44

e.g. For a correct order of operations so requires an attempt to apply g (1.5) first and then f to their g (1.5)

Also allow for an attempt to substitute x = 1.5 into $9-\left(\dfrac{3}{2x+1}\right)^2$ condoning slips such as substituting $x =$ 1.5 into $9-\dfrac{3}{(2x+1)^2}$

(c)

M1: Changes the subject of $y=\dfrac{3}{2x+1}$ and obtains $x=\dfrac{3\pm y}{2y}$ or $x=\dfrac{3}{2y}\pm\dfrac{1}{2}$ or equivalent.

Alternatively changes the subject of $x=\dfrac{3}{2y+1}$ and obtains $y=\dfrac{3\pm x}{2x}$ or $y=\dfrac{3}{2x}\pm\dfrac{1}{2}$ or equivalent.

A1: $\mathrm{g}^{-1}(x)=\dfrac{3-x}{2x}$, $\mathrm{g}^{-1}(x)=\dfrac{1}{2}\left(\dfrac{3-x}{x}\right)$, $\mathrm{g}^{-1}(x)=\dfrac{3}{2x}-\dfrac{1}{2}$, $\mathrm{g}^{-1}: x \mapsto \dfrac{3-x}{2x}$ or $\mathrm{g}^{-1}: x \mapsto \dfrac{3}{2x}-\dfrac{1}{2}$

Condone $y=\dfrac{3-x}{2x}$ o.e and even $\mathrm{g}^{-1}=\dfrac{3-x}{2x}$ but NOT $\mathrm{f}^{-1}=\dfrac{3-x}{2x}$ o.e. ISW after a correct answer.

Don't for this mark allow fractions/fractions that may be misinterpreted such as $\mathrm{g}^{-1}(x)=\dfrac{\frac{3-x}{x}}{2}$ but answers such as $\mathrm{g}^{-1}(x)=\dfrac{\frac{3}{x}-1}{2}$ and $y=\dfrac{\left(\frac{3-x}{x}\right)}{2}$ are clear and unambiguous and can score both marks.

B1: Correct domain. Allow equivalent notation e.g. $x \in (0, 3]$ but not just $(0, 3]$

Question Number	Scheme	Marks
2(a)	$R=\sqrt{5}$	B1
	$\tan\alpha=\frac{2}{1}\Rightarrow\alpha=...$	M1
	$\alpha=1.107$	A1
		(3)
(b)(i)	$\text{Max}=3+7\sqrt{5}$	B1ft
(b)(ii)	$(2x-"1.107")=\pi\Rightarrow x=...$	M1
	$\Rightarrow x=\frac{\pi+"1.107"}{2}=2.12$	A1
		(3)
		Total 6

(a)
B1: Correct exact value (Condone $R=\pm\sqrt{5}$).
isw after a correct answer. e.g. $R=\sqrt{5}=2.24$
M1: Allow for: $\tan\alpha=\pm\frac{2}{1}$, $\tan\alpha=\pm\frac{1}{2}$, $\cos\alpha=\pm\frac{1}{"R"}$, $\sin\alpha=\pm\frac{2}{"R"}$ leading to a value for α
If no method is shown imply by the sight of awrt 1.1 rads or awrt $63°$
A1: awrt 1.107
(b)(i)
B1ft: Award for 3 + 7×their R where R > 0.
Also follow through on decimal answers from (a) e.g.
Condone solutions such as $3-7\times-\sqrt{5}=18.65..$.
(b)(ii)
M1: For an attempt to solve $(2x\pm"1.107")=\pi$ or $(2x\pm"1.107")=-\pi$. May be implied by awrt 2.12
Condone a bracketing slip $\cos(2(x\pm"1.107"))=-1\Rightarrow 2(x\pm"1.107")=\pi\Rightarrow x=...$ (condone $-\pi$ as above)
Also condone an attempt to solve $(2x\pm"63°")=180°$ but not mixed units, e.g. $(2x\pm"1.107")=180°$
A1: For awrt 2.12. This cannot be given in a list, the 2.12 must be selected.

Question Number	Scheme	Marks
3(a)	$\log_{10} y = \frac{5}{16}x + 1.5$	M1A1
		(2)
(b)	$\log_{10} y = "\frac{5}{16}"x + "1.5" \Rightarrow y = 10^{"\frac{5}{16}"x + "1.5"}$	M1
	$\Rightarrow y = 10^{"\frac{5}{16}"x} \times 10^{"1.5"}$	M1
	$y = 31.6 \times 2.05^x$	A1
		(3)
		Total 5

(a)

M1: Scored for a complete attempt to get the equation of the line condoning $\log_{10} y \leftrightarrow y \leftrightarrow l$ and an incorrect sign on the gradient. So, allow for $(\log_{10} y) = \pm\frac{1.5}{4.8}x + 1.5$ o.e. and for $\frac{y-0}{x+4.8} = \pm\frac{1.5}{4.8}$

If this is attempted via simultaneous equations the mark is scored when the candidate reaches $m = \pm 0.3125$ $c = 1.5$

A1: Correct equation. e.g. $\log_{10} y = \frac{5}{16}x + 1.5$ or equivalent such as $16\log_{10} y = 5x + 24$

The $\log_{10}$ must not appear as "ln" but allow as "log" or "lg"

(b)

Main Method: Starting with their $\log_{10} y = mx + c$

M1: "Removes" the logs in their equation. e.g. $\log_{10} y = "m"x + "c" \Rightarrow y = 10^{"m"x + "c"}$

M1: "Correct" strategy to obtain values of *k* and *b* or else proceeding correctly to a form $y = 10^{"\frac{5}{16}"x} \times 10^{"1.5"}$

Allow for $k = 10^{"1.5"} (= 31.6)$ and $b = 10^{"\frac{5}{16}"} = (2.05)$. Note that you may see $10^{1.5}$ as $10\sqrt{10}$

A1: Correct equation produced, $y = 31.6 \times 2.05^x$, and no errors seen.

Condone correct working followed by $k = 31.6, b = 2.05$ with the equation being implied

cso ....the values must be 31.6 and 2.05, not values rounding to these numbers or exact values like $10\sqrt{10}$.

Note: A solution may be fudged from working similar to the following

$$\log_{10} y = mx + c \Rightarrow y = 10^{mx} + 10^{c} \Rightarrow y = 31.6 + 2.05^x \Rightarrow y = 31.6 \times 2.05^x$$

This will score Special case: M0 M1 A0

Alternative Method: Starting with $y = kb^x$

M1: Takes logs of both sides and applies at least one correct log law

e.g. $\log_{(10)} kb^x = \log_{(10)} k + \log_{(10)} b^x$, $\log_{(10)} b^x = x\log_{(10)} b$

M1: "Correct" strategy to obtain values for *k* and *b* from their $y = mx + c$

So $\log_{10} k = "c" \Rightarrow k = 10^{c}$ and $\log_{10} b = "m" \Rightarrow b = 10^{m}$

A1: Correct equation produced $y = 31.6 \times 2.05^x$ and no errors seen.
Condone correct working followed by $k = 31.6, b = 2.05$ with the equation being implied
cso ....the values must be 31.6 and 2.05, not values rounding to these numbers.
Correct answers for *k* and *b* without any working scores M0 M1 A0. Instructions on the paper state that they should show sufficient working to make their method clear.

Question Number	Scheme	Marks
4(a)	Any correct constant, so for $A=2$ or $B=3$ or $C=-1$ or $D=5$	B1
	$2x^4+15x^3+35x^2+21x-4=Ax^2(x+3)^2+Bx(x+3)^2+C(x+3)^2+D$ $\Rightarrow A=...,B=...,C=...,D=...$ or $2x^4+15x^3+35x^2+21x-4\div(x^2+6x+9)=...x^2+...x+...+\dfrac{...}{(x+3)^2}$	M1
	2 correct of $A=2,B=3,C=-1,D=5$	A1
	$A=2,B=3,C=-1,D=5$	A1
		(4)
(b)	$\int f(x)\,dx=\int\left(2x^2+3x-1+\dfrac{5}{(x+3)^2}\right)dx=\dfrac{2x^3}{3}+\dfrac{3x^2}{2}-x-\dfrac{5}{x+3}(+c)$	M1A1ftA1
		(3)
		Total 7

(a)

B1: One correct constant or one correct term in $Ax^2+Bx+C+\dfrac{D}{(x+3)^2}$

M1: Complete method for finding *A*, *B*, *C* and *D*
For example substitution/comparing coefficients/long division
Via substitution/comparing coefficients the minimum required is an identity of the correct form (condoning slips) followed by values for *A*, *B*, *C* and *D*.
See scheme but there are other versions including
$2x^4+15x^3+35x^2+21x-4=(Ax^2+Bx+C)(x+3)^2+D \Rightarrow A=...,B=...,C=...,D=...$
Via division look for a divisor of x^2+6x+9, a quotient that is a quadratic and a remainder that is either linear or a constant term.
It could be attempted by dividing by $(x+3)$ twice.
FYI, the first division gives $2x^3+9x^2+8x-3$ with a remainder of 5
A1: 2 correct constants following the award of M1
For division, when the remainder is a linear term, it would be scored for two correct of Ax^2+Bx+C
A1: All correct following the award of M1
(b)
M1: $\displaystyle\int\frac{D}{(x+3)^2}\,dx\to\frac{k}{x+3}$ where *k* is a constant.

This may be awarded following a term of $\displaystyle\int\frac{\alpha x+D}{(x+3)^2}\,dx$ following division.

Look for $\displaystyle\int\frac{\alpha x+D}{(x+3)^2}\,dx$ being correctly split and $\displaystyle\to\int\frac{\alpha x}{(x+3)^2}\,dx+\int\frac{D}{(x+3)^2}\,dx\to\text{something}+\frac{k}{x+3}$

A1ft: $\int\left(Ax^2+Bx+C+\frac{D}{(x+3)^2}\right)\mathrm{d}x=\frac{Ax^3}{3}+\frac{Bx^2}{2}+Cx-\frac{D}{x+3}(+c)\cdot$

Correct integration following through on their non-zero constants.
Allow this to be scored with *A*, *B*, *C* and *D* as above or with made up values
A1: All correct with or without "+ *c*". Allow $-1x$ for $-x$

Question Number	Scheme	Marks
5(a) Way One	$\cot^2 x - \tan^2 x \equiv \dfrac{\cos^2 x}{\sin^2 x} - \dfrac{\sin^2 x}{\cos^2 x} \equiv \dfrac{\cos^4 x - \sin^4 x}{\sin^2 x \cos^2 x}$	M1
	$\equiv \dfrac{\left(\cos^2 x - \sin^2 x\right)\left(\cos^2 x + \sin^2 x\right)}{\sin^2 x \cos^2 x} \equiv \dfrac{\cos 2x}{...}$ or $\dfrac{...}{\left(\dfrac{1}{2}\sin 2x\right)^2}$	dM1
	$\equiv \dfrac{\cos 2x}{\left(\dfrac{1}{2}\sin 2x\right)^2}$	A1
	$\equiv 4\dfrac{\cos 2x}{\sin 2x \sin 2x} \equiv 4\cot 2x \operatorname{cosec} 2x$ *	A1*
		(4)
(b)	$4\cot 2\theta \operatorname{cosec} 2\theta = 2\tan^2\theta \Rightarrow \cot^2\theta - \tan^2\theta = 2\tan^2\theta \Rightarrow \cot^2\theta - 3\tan^2\theta = 0$	M1
	$\cot^2\theta - 3\tan^2\theta = 0 \Rightarrow \dfrac{1}{\tan^2\theta} - 3\tan^2\theta = 0 \Rightarrow \tan^4\theta = \dfrac{1}{3}$	A1
	$\tan^4\theta = "\dfrac{1}{3}" \Rightarrow \tan\theta = \pm\sqrt[4]{"\dfrac{1}{3}"} = "\pm 0.7598.." \Rightarrow \theta = ...$	M1
	$\theta = \text{awrt } 0.65,\ -0.65$	A1A1
		(5)
		Total 9

(a) Way One LHS to RHS

M1: Changes the LHS to sin x and cos x and attempts to make a single fraction using a correct common denominator. Condone errors/slips on the numerator

dM1: Attempts/ applies

- Either $\cos^4 x - \sin^4 x = \left(\cos^2 x - \sin^2 x\right)\left(\cos^2 x + \sin^2 x\right) = \cos^2 x - \sin^2 x = \cos 2x$ on the numerator
- Or $\sin 2x = 2\sin x \cos x$ to the denominator condoning bracketing slip

A1: Applies both of the above correctly to achieve a correct expression in terms of cos2x and sin2x

A1*: Reaches the right hand side with sufficient working shown. Expect to see $\sin^2 2x$ split into $\sin 2x \sin 2x$

Penalise consistent (not once or twice) use of poor notation on this mark only.

Examples $\cos^2 x \leftrightarrow \cos x^2$, $\sin^2 \leftrightarrow \sin^2 x$, constantly switching between $x \leftrightarrow \theta$

(b) Allow use of $x \leftrightarrow \theta$

M1: Uses part (a) and attempt to collects terms. (See Appendix III for ways not using part (a))

See below for equations in $\sin\theta$ or $\cos\theta$ where this mark is awarded for an equation in just $\sin\theta$ or $\cos\theta$

A1: Reaches a correct equation in a single term, usually $\tan\theta$. Look for $\tan^4\theta = \dfrac{1}{3}$ o.e. such as $3\tan^4\theta = 1$

Other correct intermediate forms are $2\sin^4\theta + 2\sin^2\theta - 1 = 0$ and $2\cos^4\theta - 6\cos^2\theta + 3 = 0$

M1: Takes the 4th root of their $\dfrac{1}{3}$ (o.e) and uses $\tan^{-1}$ (you may need to check) to obtain at least one value for θ

For the other intermediate forms look for working such as $\sin^2\theta = \dfrac{\sqrt{3}-1}{2} \Rightarrow \sin\theta = \sqrt{\dfrac{\sqrt{3}-1}{2}} \Rightarrow \theta = ...$

A1: Either awrt 0.65 or awrt –0.65. Allow either answer in degrees, so awrt $\pm 37.2°$
A1: Both answers in radians, awrt ± 0.65, and no extras in range
There are many different ways to do part (a). Generally, this is how they will be marked.
Most cases can be aligned to one of the three cases.

RHS to LHS

(a) Way 2	$4\cot 2x \operatorname{cosec}2x \equiv 4\dfrac{\cos 2x}{\sin 2x}\times\dfrac{1}{\sin 2x} \equiv \dfrac{4\left(\cos^2 x - \sin^2 x\right)}{\ldots}$ or $\dfrac{\ldots}{4\sin^2 x\cos^2 x}$	M1
	$\dfrac{\cos^2 x - \sin^2 x}{\sin^2 x\cos^2 x} \equiv \dfrac{1}{\sin^2 x} - \dfrac{1}{\cos^2 x} \equiv \operatorname{cosec}^2 x - \sec^2 x$	dM1A1
	$\equiv 1+\cot^2 x - 1 - \tan^2 x \equiv \cot^2 x - \tan^2 x$ *	A1*

M1: Changes to sin 2*x* and cos 2*x* or tan2*x* and sin2*x* and attempts single angles in sin*x* and cos*x*
dM1: Changes to single angles throughout and splits into 2 separate fractions (which don't need to be simplified)
A1: Correct expression in terms of the single angles cosec x and sec x
A1*: Reaches the left hand side with sufficient working shown

Working on both sides: One possible way

(a) Way 3	$\cot^2 x - \tan^2 x \equiv 4\cot 2x \operatorname{cosec}2x$ $\dfrac{\cos^2 x}{\sin^2 x} - \dfrac{\sin^2 x}{\cos^2 x} \equiv 4\times\dfrac{\cos 2x}{\sin 2x}\times\dfrac{1}{\sin 2x}$ $\cos^4 x - \sin^4 x \equiv 4\sin^2 x\cos^2 x\dfrac{\cos 2x}{\sin 2x}\times\dfrac{1}{\sin 2x}$	M1
	$\cos^4 x - \sin^4 x \equiv 4\times\sin^2 x\cos^2 x\dfrac{\left(\cos^2 x - \sin^2 x\right)}{\left(2\sin x\cos x\right)^2}$	dM1A1
	$\left(\cos^4 x - \sin^4 x\right) \equiv \left(\cos^2 x - \sin^2 x\right)$ $\left(\cos^2 x - \sin^2 x\right)\underset{1}{\cancel{\left(\cos^2 x + \sin^2 x\right)}} \equiv \left(\cos^2 x - \sin^2 x\right)$ Hence true	A1*

M1: Changes to sin *x*, cos *x*, sin 2*x* and cos2 *x* and attempts to cross multiply
dM1: Applies

- either $\cos^2 x - \sin^2 x = \cos 2x$ to the numerator
- or $\sin 2x = 2\sin x \cos x$ to the denominator

A1: Correct identity in terms of cos x and sin x
A1*: Reaches a point where both sides are equal and makes a minimal comment
Example of how you mark a "different" approach.

Question Number	Scheme	Marks
6(a)	(i) $P(0,\ 3a)$	B1
	(ii) $Q(a,0)\quad R\left(\frac{7}{3}a,0\right)$	B1 B1
	(iii) $S\left(\frac{5}{3}a,\ -2a\right)$	B1
		(4)
(b)	$3x-5a-2a=x-2a\Rightarrow x=...$ **or** $-(3x-5a)-2a=-(x-2a)\Rightarrow x=...$	M1
	$x=\frac{5}{2}a$ **or** $x=\frac{1}{2}a$	A1
	$3x-5a-2a=x-2a\Rightarrow x=...$ **and** $-(3x-5a)-2a=-(x-2a)\Rightarrow x=...$	dM1
	$x=\frac{5}{2}a$ **and** $x=\frac{1}{2}a$	A1
		(4)
		Total 8

(a) Question states simplest form so don't accept e.g. $\frac{10}{6}a$ for $\frac{5}{3}a$

(i)
B1: For (0, 3a). Condone just $y = 3a$ as long as there isn't a value for x coordinate (apart from 0).
Note that either of $P = 3a$ or (3a, 0) is B0 unless y =3a is previously seen
(ii)
B1: For either coordinate (a, 0) **or** $\left(\frac{7}{3}a,\ 0\right)$ which may or may not be linked correctly to Q and R

Condone just $x = a$ (x = 1a) or $x=\frac{7}{3}a$ as long as there isn't a y coordinate (that isn't 0).

Note that an answer such as $Q = a$ or (0, a) is B0 unless $x = a$ is previously seen.

Likewise an answer such as $R=\frac{7}{3}a$ or (0,7 a/ 3) is B0 unless x =7 a/3 is previously seen

B1: For (a, 0) **and** $\left(\frac{7}{3}a,\ 0\right)$. Allow the coordinates to be given separately. They do not need to be linked to the correct letter. Condone just $x = a$ (x = 1a) and $x=\frac{7}{3}a$ as long as there isn't a y coordinate(that isn't 0).

SC: $Q = a$ **AND** $R=\frac{7}{3}a$ or vice versa is B1 B0

(iii)
B1: $\left(\frac{5}{3}a,\ -2a\right)$. Allow this to be given as x = ..., y = ...

(b)
M1: Attempts to solve one **correct** equation. It must be an equation without moduli and be found correctly
For example, look for

- Either taking both positive aspects of the modulus equation $|3x-5a|-2a=|x-2a|$

  So $3x-5a-2a=x-2a$ or exact equivalent such as $3x-7a=x-2a \Rightarrow x=...$

  Condone for this mark $\Rightarrow a=...$

- Or taking both negative aspects of the modulus equation $|3x-5a|-2a=|x-2a|$

  So $-3x+5a-2a=-x+2a$ or exact equivalent such as $-3x+3a=-x+2a \Rightarrow x=...$

  Condone for this mark $\Rightarrow a=...$

  The $2a$ may be moved over first so it is possible to solve $|3x-5a|=|x-2a|+2a$

  So both positive will give an equation $3x-5a=x-2a+2a$ or $3x-5a=x$

  or both negative would give $-3x+5a=-x+2a+2a$ or $-3x+5a=-x+4a$

A1: One correct value of x which must be found from a correctly produced equation
dM1: Attempts to solve both of these equations.
There may be other equations which should be ignored for this mark
A1: Both correct values of x (coming from correct equations) with no other values given.
Also allow if 4 answers are found, for example, followed by the 2 correct answers being chosen and the other 2 incorrect answers discarded.

Question Number	Scheme	Marks
7(a)	$x = 3\tan\left(y-\dfrac{\pi}{6}\right) \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = 3\sec^2\left(y-\dfrac{\pi}{6}\right)$	B1
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{3\sec^2\left(y-\dfrac{\pi}{6}\right)}$	M1
	$\dfrac{1}{3\left(1+\tan^2\left(y-\dfrac{\pi}{6}\right)\right)} = \dfrac{1}{3\left(1+\left(\dfrac{x}{3}\right)^2\right)}$	dM1
	$= \dfrac{3}{x^2+9}$	A1
		(4)
(b)	$y = \dfrac{\pi}{3} \Rightarrow x = 3\tan\dfrac{\pi}{6} = \sqrt{3}$	B1
	$x = \sqrt{3} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3}{\left(\sqrt{3}\right)^2+9}$ or $y = \dfrac{\pi}{3} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{3\sec^2\left(\dfrac{\pi}{3}-\dfrac{\pi}{6}\right)} = \ldots$	M1
	$y - \dfrac{\pi}{3} = \dfrac{1}{4}\left(x-\sqrt{3}\right)$	dM1
	$y = 0 \Rightarrow 0 - \dfrac{\pi}{3} = \dfrac{1}{4}\left(x-\sqrt{3}\right) \Rightarrow x = \ldots$	ddM1
	$x = \sqrt{3} - \dfrac{4\pi}{3}$	A1
		(5)
		Total 9

(a)

B1: Correct derivative, including correct lhs. Condone $\dfrac{\mathrm{d}x}{\mathrm{d}y} = k\sec^2\left(y-\dfrac{\pi}{6}\right)$ where k is a constant.

M1: Either (1) attempts to apply the reciprocal rule $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1 \div \dfrac{\mathrm{d}x}{\mathrm{d}y}$.

Don't be too concerned by the position of the constant, it is the function and variable that is important.

Or (2) attempts to apply $\sec^2\left(y-\dfrac{\pi}{6}\right) = 1 + \tan^2\left(y-\dfrac{\pi}{6}\right)$ with $\tan\left(y-\dfrac{\pi}{6}\right)$ being replaced by $\dfrac{x}{3}$ to get $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x

dM1: Attempts both (1) and (2) to obtain $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x

A1: Correct answer

(b)

B1: Correct value for x. Condone awrt 1.73. This can be awarded from the sight of this value in an equation

M1: Uses a correct method to find the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ which may be a decimal.

Note that this can be scored either from answer to part (a) e.g $x = "\sqrt{3}" \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = "\dfrac{3}{\left(\sqrt{3}\right)^2 + 9}"$

Or via their $\dfrac{\mathrm{d}x}{\mathrm{d}y} = 3\sec^2\left(\dfrac{\pi}{3} - \dfrac{\pi}{6}\right)$ followed by an attempt at the reciprocal

dM1: Correct straight line method for the tangent at $\left("\sqrt{3}", \dfrac{\pi}{3}\right)$ using their correctly found *m*.

It is dependent upon having found the gradient and *x* value using a correct method.
If they use the form $y = mx + c$ they must proceed as far as *c* = ...

ddM1: Uses *y* = 0 to find *x*. It is dependent upon having scored the previous mark

A1: Correct value or exact equivalent for example $\dfrac{3\sqrt{3} - 4\pi}{3}$

Alt (a) via arctan

B1: $x = 3\tan\left(y - \dfrac{\pi}{6}\right) \Rightarrow y = \dfrac{\pi}{6} + \arctan\left(\dfrac{x}{3}\right) \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 + \dfrac{1}{1 + \left(\dfrac{x}{...}\right)^2} \times ...$ where ... could be any values even 1

M1: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 + \dfrac{1}{1 + \left(\dfrac{x}{3}\right)^2} \times ...$ where ... could be any value even 1

dM1: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 + \dfrac{1}{1 + \left(\dfrac{x}{3}\right)^2} \times \dfrac{1}{3}$

A1*: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3}{x^2 + 9}$

Alt (a) via compound angle identity they could pick up the first two marks

$$x = \frac{3\left(\tan y - \tan\dfrac{\pi}{6}\right)}{1 + \tan\dfrac{\pi}{6}\tan y} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = \frac{\left(1 + \tan\dfrac{\pi}{6}\tan y\right) \times 3\sec^2 y - 3\left(\tan y - \tan\dfrac{\pi}{6}\right) \times \tan\dfrac{\pi}{6}\sec^2 y}{\left(1 + \tan\dfrac{\pi}{6}\tan y\right)^2}$$

$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\left(1 + \tan\dfrac{\pi}{6}\tan y\right)^2}{\left(1 + \tan\dfrac{\pi}{6}\tan y\right) \times 3\sec^2 y - 3\left(\tan y - \tan\dfrac{\pi}{6}\right) \times \tan\dfrac{\pi}{6}\sec^2 y}$$

Question Number	Scheme	Notes
8	$\int (2\cos x - \sin x)^2 \, dx = \int (4\cos^2 x - 4\sin x \cos x + \sin^2 x) \, dx$	M1
	$\int 4\sin x \cos x \, dx = \int 2\sin 2x \, dx = -\cos 2x$ or $\int 4\sin x \cos x \, dx = -2\cos^2 x$ or $2\sin^2 x$	M1
	$\int (4\cos^2 x + \sin^2 x) \, dx = \int (1 + 3\cos^2 x) \, dx = \int \left(1 + 3\left(\frac{\cos 2x + 1}{2}\right)\right) dx$ or $\int (4\cos^2 x + \sin^2 x) \, dx = \int \left(4\left(\frac{\cos 2x + 1}{2}\right) + \frac{1 - \cos 2x}{2}\right) dx$	M1
	$\int (2\cos x - \sin x)^2 \, dx = \frac{3}{4}\sin 2x + \cos 2x + \frac{5}{2}x (+c)$ or $\int (2\cos x - \sin x)^2 \, dx = \frac{3}{4}\sin 2x + 2\cos^2 x + \frac{5}{2}x (+c)$ or $\int (2\cos x - \sin x)^2 \, dx = \frac{3}{4}\sin 2x - 2\sin^2 x + \frac{5}{2}x (+c)$	A1A1
		(5)
		Total 5

Condone changes in variables throughout this solution as long as the answer is given in terms of x

M1: Expands to the form $p\cos^2 x + q\sin x \cos x + r\sin^2 x$

M1: Correct strategy for **integrating** $q\sin x \cos x$ (i.e. obtains $k\cos 2x$ or $k\sin^2 x$ or $k\cos^2 x$)

M1: Correct strategy for rewriting $p\cos^2 x + r\sin^2 x$ **into a form that can be integrated**.

Score for one of

- writes in terms of just $\sin^2 x$ and then uses $\sin^2 x = \frac{\pm 1 \pm \cos 2x}{2}$
- writes in terms of just $\cos^2 x$ and then uses $\cos^2 x = \frac{\pm 1 \pm \cos 2x}{2}$
- writes both $\sin^2 x$ and $\cos^2 x$ in terms of $\cos 2x$ with **at least one** of these via use of a correct/allowable form. That is $\sin^2 x = \frac{\pm 1 \pm \cos 2x}{2}$ or $\cos^2 x = \frac{\pm 1 \pm \cos 2x}{2}$

A1: Integrates and achieves 2 correct terms (of the 3 required terms)

NB: An unsimplified expression is acceptable for this mark so please check carefully. e.g. $2x + ... + \frac{1}{2}x$ counts as one correct term.

A1: Correct simplified integration (+ c not required).

An alternative solution is via "R cos" or "R sin" but the last two marks are unlikely to be achieved due to the fact that an exact answer is difficult to arrive at.

Question Number	Scheme	Notes
8	$\int (2\cos x - \sin x)^2 \,dx = \int 5\cos^2(x+0.464)\,dx$	M1
	$= \int \frac{5\{\cos(2x+0.927)+1\}}{2}\,dx$	M1
	$\frac{5\sin(2x+0.927)}{4}+\frac{5}{2}x$	M1
	$\frac{3\sin 2x}{4}+\cos 2x+\frac{5}{2}x+c$	A1A1
		(5)
		Total 5

M1: Writes $2\cos x - \sin x$ in the form $R\cos(x \pm \alpha)$ o.e. and squares.

Requires a full method so for the form $R\cos(x \pm \alpha)$

- requires $R^2 = 2^2 + 1^2$
- requires $\tan\alpha = \pm\frac{1}{2} \Rightarrow \alpha = ...$ in radians but condone in degrees

M1: Correct strategy for writing $\cos^2(x \pm \alpha)$ into a form that can be integrated using the double angle formula

$\cos^2(x \pm \alpha) \to \frac{\pm 1 \pm \cos(2x \pm 2\alpha)}{2}$ but condone $\cos^2(x \pm \alpha) \to \frac{\pm 1 \pm \cos(2x \pm \alpha)}{2}$

α should be in radians but condone in degrees

M1: Correct strategy for writing $\cos^2(x \pm \alpha)$ into a form that can be integrated using the double angle formula

$\cos^2(x \pm \alpha) \to \frac{\pm 1 \pm \cos(2x \pm 2\alpha)}{2}$ but condone $\cos^2(x \pm \alpha) \to \frac{\pm 1 \pm \cos(2x \pm \alpha)}{2}$

α should be in radians but condone in degrees

M1: It is for integrating $\cos(2x \pm \delta) \to \pm\sin(2x \pm \delta)$ following use of an acceptable double angle formula

A1: 2 correct terms of $\frac{3\sin 2x}{4}+\cos 2x+\frac{5}{2}x+c$

A1: Fully correct and simplified (+ c not required) or $\frac{5\sin(2x+2\arctan 0.5)}{4}+\frac{5}{2}x$

Question Number	Scheme	Notes
8	$\int (2\cos x - \sin x)^2 \, dx = \int 5\sin^2 (x - 1.107) \, dx$	M1
	$= \int \frac{5\{1 - \cos(2x - 2.21)\}}{2} \, dx$	M1
	$= \frac{5}{2}x - \frac{5\sin(2x - 2.21)}{4}$	M1
	$= \frac{5}{2}x + \frac{3\sin 2x}{4} + \cos 2x + c$	A1A1
		(5)
		Total 5

Question Number	Scheme	Marks
9(a)	$y=\sqrt{3+4e^{x^2}}=\left(3+4e^{x^2}\right)^{\frac{1}{2}}\Rightarrow\frac{dy}{dx}=\frac{1}{2}\left(3+4e^{x^2}\right)^{-\frac{1}{2}}\times 8xe^{x^2}$	M1
	$=4xe^{x^2}\left(3+4e^{x^2}\right)^{-\frac{1}{2}}$	A1
		(2)
(b)	$\frac{\left(3+4e^{x^2}\right)^{\frac{1}{2}}}{x}=4xe^{x^2}\left(3+4e^{x^2}\right)^{-\frac{1}{2}}$	M1
	$\frac{\left(3+4e^{x^2}\right)}{x}=4xe^{x^2}$	dM1
	$4x^2e^{x^2}-4e^{x^2}-3=0*$	A1*
		(3)
(c)	$f(x)=4x^2e^{x^2}-4e^{x^2}-3\Rightarrow f(1)=-3$ AND $f(2)=652\cdot...$	M1
	Change of sign and f (x) is continuous hence root in (1, 2)	A1
		(2)
(d)	$4x^2e^{x^2}-4e^{x^2}-3=0\Rightarrow x^2=\frac{4e^{x^2}+3}{4e^{x^2}}=\frac{4+3e^{-x^2}}{4}\Rightarrow x=\frac{1}{2}\sqrt{4+3e^{-x^2}}\ *$	B1*
		(1)
(e)(i)	$x_1=1\Rightarrow x_2=\frac{1}{2}\sqrt{4+3e^{-1}}$	M1
	$x_3=1.0997$	A1
(ii)	$\alpha=1.1051$	A1
		(3)
		Total 11

(a)

M1: Differentiates using the chain rule to obtain $kxe^{x^2}\left(3+4e^{x^2}\right)^{-\frac{1}{2}}$ OR $k\,e^{x^2}\left(3+4e^{x^2}\right)^{-\frac{1}{2}}$

If they are using implicit differentiation accept $\frac{kxe^{x^2}}{y}$ or $\frac{ke^{x^2}}{y}$

A1: Correct derivative in simplest form. Also award for $\frac{dy}{dx}=\frac{4xe^{x^2}}{y}$ o.e. simplified answer

(b)

M1: Sets their $\frac{dy}{dx}=\frac{\left(3+4e^{x^2}\right)^{\frac{1}{2}}}{x}$ or equivalent such as use of $y=mx$ with $m=$ their $\frac{dy}{dx}$ and $y=\left(3+4e^{x^2}\right)^{\frac{1}{2}}$.
Allow with another variable $x\to\alpha$

dM1: Multiplies up to eliminate the square root. Allow a consistent use of another variable $x\to\alpha$
Dependent upon the previous M1 and a suitable $\frac{dy}{dx}$, that is one of the form that scored an M1 in part (a)

A1*: Correct proof

An alternative method may be seen using the equation of the line with gradient $\frac{\mathrm{d}y}{\mathrm{d}x}$ and point $\left(\alpha,\sqrt{3+4\mathrm{e}^{\alpha^2}}\right)$

M1: For $y-\sqrt{3+4\mathrm{e}^{\alpha^2}}=\text{"}\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=\alpha}\text{"}(x-\alpha)$ and sets $(x,y)=(0,0)$

dM1: Proceeds as the main method with the same condition on the gradient

A1*: Correct proof. Condone with variable α

(c)

M1: Attempts both f (1) and f (2) (or tighter) and obtains at least one value correct to 1sf rounded or truncated.

Condone f (2) = $12\mathrm{e}^4-3$ If f or y is not stated assume that f (x) = $4x^2\mathrm{e}^{x^2}-4\mathrm{e}^{x^2}-3$

A1: Requires

- both values either correct or rounded/ truncated with accuracy to at least 1 sf
- a reference to a sign change e.g. $\mathrm{f}(1)=-3<0$, $\mathrm{f}(2)=652>0$ or f(1) × f(2) <0
- a mention of continuity
- a minimal conclusion, e.g. hence root

(d)

B1*: Correct proof.

Look for the following evidence

- the line $4x^2\mathrm{e}^{x^2}-4\mathrm{e}^{x^2}-3=0$ but may be implied by $4x^2\mathrm{e}^{x^2}=4\mathrm{e}^{x^2}+3$ o.e.
- the line $x^2=\frac{4\mathrm{e}^{x^2}+3}{4\mathrm{e}^{x^2}}$ o.e $x^2=1+\frac{3}{4\mathrm{e}^{x^2}}$ or $4x^2=\frac{4\mathrm{e}^{x^2}+3}{\mathrm{e}^{x^2}}$ o.e $4x^2=4+\frac{3}{\mathrm{e}^{x^2}}$
- correct square root work leading to the given answer

(e)

M1: Attempts to use the iteration formula.

This may be implied by awrt 1.13 for x_2 or awrt 1.10 for x_3 or sight of a correctly embedded value.

It cannot be awarded for just the value of α

A1: awrt 1.0997

A1: 1.1051 cao following the award M1

Question Number	Scheme	Marks
10(a)	$(F=)35$	B1
		(1)
(b)	$200=\dfrac{350e^{15k}}{9+e^{15k}} \Rightarrow 1800+200e^{15k}=350e^{15k} \Rightarrow 150e^{15k}=1800$	M1
	$e^{15k}=\dfrac{1800}{150} \Rightarrow 15k=\ln 12 \Rightarrow k=\dfrac{1}{15}\ln 12*$	dM1A1*
		(3)
(c)	$F=\dfrac{350e^{kt}}{9+e^{kt}} \Rightarrow \dfrac{dF}{dt}=\dfrac{350ke^{kt}\left(9+e^{kt}\right)-350e^{kt}\left(ke^{kt}\right)}{\left(9+e^{kt}\right)^2}$	M1A1
	$\dfrac{3150ke^{kt}}{\left(9+e^{kt}\right)^2}=10 \Rightarrow 315ke^{kt}=81+18e^{kt}+e^{2kt} \Rightarrow e^{2kt}+(18-315k)e^{kt}+81=0$	M1
	$e^{2kt}+(18-315k)e^{kt}+81=0 \Rightarrow e^{kt}=\dfrac{315k-18\pm\sqrt{(18-315k)^2-4\times 81}}{2} \Rightarrow kt=...$	M1
	$T=$ awrt $5.7,\ 20.8$	A1
		(5)
		Total 9

(a)
B1: Correct value, 35
(b)
M1: Uses F = 200 and t = 15 and reaches $Ae^{15k}=B$ where $A\times B>0$
dM1: Proceeds using correct order of operations to obtain a value for k

Look for $Ae^{15k}=B \Rightarrow e^{15k}=\dfrac{B}{A} \Rightarrow 15k=\ln\dfrac{B}{A} \Rightarrow k=...$

or Alternatively $Ae^{15k}=B \Rightarrow \ln A+15k=\ln B \Rightarrow 15k=\ln B-\ln A \Rightarrow k=...$

A1*: Correct proof with all necessary steps shown.

Minimum acceptable solution $200=\dfrac{350e^{15k}}{9+e^{15k}} \Rightarrow 150e^{15k}=1800 \Rightarrow e^{15k}=12 \Rightarrow k=\dfrac{1}{15}\ln 12$

(c) Allow the whole of part (c) to be done with the letter k, the exact value for k or using k = awrt 0.166
M1: Correct attempt of the quotient (product or chain) rule.

For the Quotient Rule look for $\dfrac{dF}{dt}=\dfrac{Pe^{kt}\left(9+e^{kt}\right)-Qe^{kt}\left(e^{kt}\right)}{\left(9+e^{kt}\right)^2}$ $\quad P,Q>0$

For the Product Rule look for $\dfrac{dF}{dt}=Pe^{kt}\left(9+e^{kt}\right)^{-1}\pm Qe^{kt}e^{kt}\left(9+e^{kt}\right)^{-2}$ $\quad P,Q>0$

For Chain Rule look for $F=A\pm\dfrac{B}{9+e^{kt}} \Rightarrow \dfrac{dF}{dt}=Qe^{kt}\left(9+e^{kt}\right)^{-2}$ $\quad Q$ is a constant

A1: Correct differentiation, which may be unsimplified.

Allow for an expression in k , with exact $k=\dfrac{1}{15}\ln 12$, or using k = awrt 0.166

M1: Sets their derivative = 10 and obtains a 3TQ in e^{kt}
It is dependent upon a reasonable attempt to differentiate.

In almost all cases the M1 will have been awarded but condone an attempt following $\dfrac{dF}{dt}=\dfrac{vu'+uv'}{v^2}$

M1: Scored for

- solving a 3TQ in e^{kt} by any method including a calculator (you may need to check with accuracy to 2sf rounded or truncated). Note that the equation may be a quadratic in $12^{\frac{1}{15}t}$ instead of $e^{\left(\frac{1}{15}\ln 12\right)t}$
- then taking ln's to obtain at least one value for kt
  FYI the correct quadratics are;
  $e^{\left(\frac{2}{15}\ln 12\right)t}+(18-21\ln 12)e^{\left(\frac{1}{15}\ln 12\right)t}+81=0,$

  or $\left(e^{0.166t}\right)^2-34.18e^{0.166t}+81=0\Rightarrow e^{0.166t}=2.56, 31.62\Rightarrow 0.166t=0.94, 3.45$

A1: awrt 5.7, 20.8